Written Exam at the Department of Economics Summer 2018

Micro III

Final Exam

June 12, 2018

(2-hour closed book exam)

Answers only in English.

This exam question consists of 3 pages in total (including the current page).

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Be careful not to cheat at exams!

- You cheat at an exam, if during the exam, you:
- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

Solution Guide - Micro III Exam, June 2018

1. Consider a first-price sealed bid auction with $N \geq 2$ bidders, who have valuations $v_1, v_2, \ldots, v_N$. Each value v_i, $i \in \{1, 2, \ldots N\}$ is independently drawn from a uniform distribution on $[0, 1]$. Values are **private**.

   (a) Show that there is a symmetric Bayesian Nash equilibrium in linear strategies, $b(v_i) = cv_i$, by explicitly solving for the value of c.

   (b) Compare your answer in part (a) to the answer you would obtain in the special case with two bidders, $N = 2$. Briefly comment on any similarities and differences (2-3 sentences).

   (c) What would happen to bidding behavior in this auction if the number of bidders became very large (say if N increased without bound)? Intuitively describe, in words, why this is the case, and what you expect would happen to the expected revenue of the seller (2-4 sentences). *Please attempt this question even if you did not successfully complete the earlier parts.*

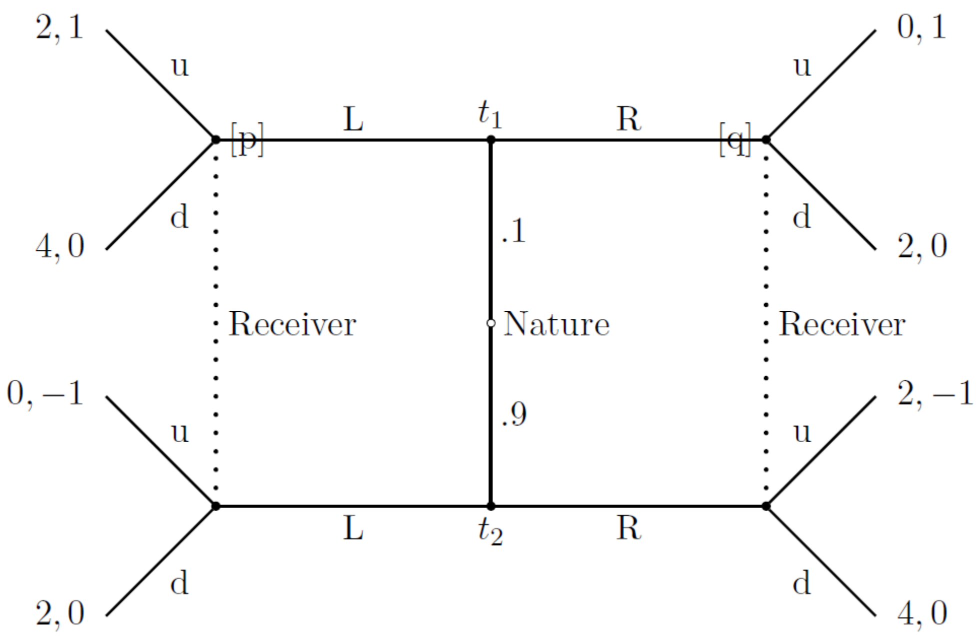
Answer - Question 1

(a) The expected payoff of bidder i from bidding b_i is $\mathbb{P}(b_i > max_{j \neq i} b(v_j))(v_i - b_i)$. This simplifies to $\mathbb{P}(\frac{b_i}{c} > v_j, \forall j \neq i)(v_i - b_i) = \left(\frac{b_i}{c}\right)^{N-1}(v_i - b) = \frac{b_i^{N-1}v_i - b_i^N}{c^{N-1}}$. The first-order-condition is $\frac{(N-1)b^{N-2}v_i - Nb_i^{N-1}}{c^{N-1}} = \left(\frac{b^{N-2}}{c^{n-1}}\right)[(N-1)v_i - Nb_i] = 0$, which implies $b_i = (\frac{N-1}{N})v_i$, i.e. $c = \frac{N-1}{N}$.

(b) Plugging in $N = 2$ to the answer in part (a) gives $c = 1/2$, so both bidders then bid half of their valuation. Similarities with part (a) include that high-valuation bidders place higher bids than low-valuation bidders, and that bidders all engage in bid shading, i.e. bidding below their valuation, in an attempt to capture positive surplus. A difference is that bidders face less competition when $N = 2$ than when $N > 2$, and so shade their bids to a greater extent.

(c) As N increases without bound, $\frac{N-1}{N}$ tends to 1, so in the limit all bidders bid their full valuation. Competition becomes so fierce that bidders no longer engage in bid shading, as they know that reducing their bid will decrease (in a relative sense) their change of winning the auction to a large degree. The seller's expected revenue in the limit would be equal to the expected valuation of the highest-valuation bidder, i.e equal to 1. Intuitively, with very many bidders, there is likely at least one bidder with valuation very close to 1, and that bidder will bid very close to his full valuation.

2. Now consider the following game G:



Note that in this game, the prior probability that the sender is of type 1 is equal to 0.1.

(a) Briefly explain whether G is a static or a dynamic game (1 sentence), and whether or not G is a cheap talk game (1 sentence).

(b) Find a separating equilibrium in G, and find a pooling equilibrium where both sender types play L.

(c) Check whether the equilibria you found in part (b) satisfy Signaling Requirement 6 ('*equilibrium domination*').

(d) Describe a real-world strategic situation that could correspond to G, and explain why this is the case (3-4 sentences). What (if anything) do your answers to parts (b) and (c) suggest about the behavior we are likely to see in this real-world situation? (2-3 sentences)

Answer - Question 2

(a) By definition, G is a dynamic game, where the Sender chooses a message and the Receiver then responds with an action. It is not a cheap talk game, because the Sender's payoff depend directly on his chosen message.

(b) Separating equilibrium: $(LR, ud, p = 1, q = 0)$. Pooling equilibria: $(LL, du, p = 0.1, q \geq 1/2)$.

(c) The separating equilibrium satisfies Signaling Requirement 6, since there are no off-the-equilibrium-path beliefs, and hence nothing to check. The pooling equilibrium does not satisfy Signaling Requirement 6; t_1's equilibrium payoff of 4 is strictly higher than any payoff he could possibly get by deviating to R, whereas t_2's equilibrium payoff of 2 is not. Signaling Requirement 6 therefore implies $q = 0$, whereas the pooling equilibrium required $q \geq 1/2$.

(d) One possible answer is as follows. The Sender could be a CEO with two possible levels of skill in dealing with competition (t_1 is low-skill, t_2 is high-skill). Each message could be a possible educational degree for the CEO, where the low-skill (high-skill) CEO finds education R (L) more costly. The Receiver could be another firm deciding whether to compete (u) or not (d) with the CEO's firm in the product market. The analysis above might suggest that the CEO takes an education level that reveals his skill, and the other firm decides to compete if and only if this skill level is revealed to be low (separating equilibrium). More generally, some key features of the game are as follows: the Sender always wants the Receiver to take the same action, the Receiver only benefits from taking that action if the Sender is of a particular type, and the Sender has different messages at his disposal that are more costly for one type than for another, but that do not directly affect the Receiver's payoff.

3. Consider a static game F where two firms produce a homogeneous good and compete in quantities. Firm 1 and Firm 2 both produce at zero cost. Let q_i denote the quantity produced by Firm $i \in \{1, 2\}$. Given q_1 and q_2, the market price is $p = 4 - q_1 - q_2$. Both firms choose quantities simultaneously, and maximize profits.

   (a) Solve for the Nash equilibrium of this game. What profits do Firm 1 and Firm 2 earn in equilibrium? What profits would Firm 1 and Firm 2 earn if they instead each produced half of the monopoly quantity (i.e. half of the quantity that maximizes total industry profits)?

   Now consider a dynamic game, with infinite time horizon, where Firm 1 and Firm 2 play the stage game F in periods $t = 1, 2, 3, \ldots$. You can assume that both firms discount future payoffs with factor $\delta \in (0, 1)$.

   (b) For what values of δ can you find a subgame perfect Nash equilibrium where, on the equilibrium path, each firm produces half of the monopoly quantity in each period? Explicitly solve for this equilibrium, making sure to state each firm's strategy, and show that neither firm has an incentive to deviate.

   Now assume that there is an anti-trust authority in this market which acts in the following way: in any period t where the total quantity produced, $q_1 + q_2$, differs from the monopoly quantity, the anti-trust authority does nothing; in any period t where the total quantity produced $q_1 + q_2$ is equal to the monopoly quantity, there is a probability $p \in [0, 1)$ that the anti-trust authority detects collusion. If collusion is detected, then the anti-trust authority cannot impose a fine, but instead forces both firms to leave the market at the end of period t (so that both firms then earn zero profits in period $t+1, t+2, \ldots$). Notice that in the special case with $p = 0$, collusion is never detected, so the anti-trust authority plays no role, and is effectively absent from the market.

   (c) Consider the dynamic game described above where the anti-trust authority can detect collusion with probability $p \in [0, 1)$. Write down an inequality, involving p and δ, which implicitly defines the values of δ for which neither firm has an incentive to deviate from the equilibrium you found in part (b) (you do **not** need to explicitly solve this inequality to isolate δ).

   (d) Using your answers in parts (b) and (c), comment on how the presence of the anti-trust authority may (or may not) impact firm behavior in this market, compared to a setting where the anti-trust authority was absent. Does your answer depend on the value of p, and on whether the value of the discount factor is low, intermediate, or high (3-4 sentences)? Briefly give the intuition behind your answer (2-3 sentences). *Please attempt this question even if you did not successfully complete the earlier parts.*

Answer - Question 3

(a) Firm i profits are $\pi_i = q_i(4 - q_i - q_j)$ for $i \in \{1, 2\}$. The first-order-condition is $4 - 2q_i - q_j = 0$, yielding a Nash equilibrium of $q_i^* = q_j^* = 4/3 \equiv q^{n.e.}$ (notice that profit functions are strictly concave, so the second-order condition is always satisfied). Each firm earns equilibrium profits of $\pi^{n.e.} = 16/9$. Industry profits, given total quantity $q_i + q_j \equiv Q$, are $\pi_i + \pi_j = Q(4 - Q)$. This implies a monopoly quantity of $Q = 2$, and a monopoly price of 2. If each firm produced half of the monopoly quantity, $q_i = q_j = 1$, then each would earn $\pi^{mon} = 2$.

(b) Consider the strategy profile $(Trigger_1, Trigger_2)$, where $Trigger_i$ for firm $i \in \{1, 2\}$ is defined as follows: 'At $t = 1$, choose $q_i = q^{mon} = 1$. At $t \geq 2$, choose $q_i = q^{mon} = 1$ if $(q_i = 1, q_j = 1)$ were the quantities chosen in all periods $t' \leq t-1$; otherwise choose $q_i = q^{n.e} = 4/3$.' Off the equilibrium path, both firms always choose the Nash equilibrium quantities of the stage game, which by definition are best responses to one another. Thus, neither firm has an incentive to deviate in subgames that are off the equilibrium path. On the equilibrium path, the relevant condition is $\frac{1}{1-\delta}\pi^{mon} \geq \pi^{dev} + \frac{\delta}{1-\delta}\pi^{n.e}$, or equivalently $\delta \geq \frac{\pi^{dev}-\pi^{mon}}{\pi^{dev}-\pi^{n.e.}}$, where π^{dev} are the profits earned by firm i playing a best response to $q_j = 1$, i.e. setting $q_i = 3/2$, to earn $\pi^{dev} = 9/4$. Direct substitution of $\pi^{mon} = 2$, $\pi^{n.e.} = 16/9$, and $\pi^{dev} = 9/4$ into the relevant inequality yields $\delta \geq \frac{9/4-2}{9/4-16/9}$, which simplifies to $\delta \geq 9/17$. Thus, this strategy profile constitutes a subgame perfect Nash equilibrium for values of the discount factor satisfying $\delta \in [9/17, 1)$.

(c) To check for deviations on the equilibrium path, the relevant inequality is now $\frac{1}{1-(1-p)\delta}\pi^{mon} \geq \pi^{dev} + \frac{\delta}{1-\delta}\pi^{n.e}$. The key point is that colluding firms now discount future profits for two reasons: because of the discount factor, but also because the game may not reach the following period (i.e. the anti-trust authority may force them to shut down) with probability p. Thus, the effective discount factor on the equilibrium path, where both firms produce half of the monopoly quantity, is $\delta(1 - p)$, compared with δ in part (b).

(d) The relevant inequality in part (c) is more difficult to satisfy than in part (b), whenever $p > 0$. Thus, for $p > 0$, the presence of the anti-trust authority in the market makes it more difficult for firms to sustain collusion. The impact of the anti-trust authority is increasing in p, in the sense that collusion is more difficult to sustain, the more likely it is to be detected. Regarding the discount factor: firms can always collude if they are sufficiently patient, and can never collude if they are sufficiently impatient. This suggests that the presence of the anti-trust authority may matter most when the discount factor is intermediate, rather than very low or very high. It is then that the authority can likely change firm behavior; by reducing the expected payoff of firms that collude, but not the expected payoff of firms that deviate from collusion or that play the stage-game Nash equilibrium.